PHASE-DEPENDENT RCS MEASUREMENTS IN THE PRESENCE OF OUTLIERS *

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ABSTRACT

Coherent radar cross section measurements on a moving target in free space will trace a circle centered on the origin of the complex (I, Q) plane. The presence of additional complex signal (for example, background, clutter, average of the target-mount interaction, etc.) that do not depend on target position will translate the origin of the circle to some complex point (I_0, Q_0).

A number of techniques have been used successfully to analyze phase-dependent I-Q data [1]. The presence of outliers, however, can degrade the results, that is, significant errors can be introduced in the determination of all 3 parameters. Hence, we seek to increase the robustness of our analysis technique by first eliminating or reducing the influence of outliers. This is especially important at sub-wavelength translations at VHF, where spectral techniques are not applicable and only a limited arc of data is available.

A combination of a robust and efficient Least-Median Square (LMS) [2,3] and an Orthogonal Distance Regression (ODR)[4] algorithm is used 1) to eliminate outliers, and then 2) to separate the target and background signals. We analyze data obtained as an Arrow III target moves relative to its supporting pylon. To demonstrate the effectiveness of the technique, we introdue rf interference signals into S band data and show that the uncontaminated parameters can be recovered.

Key words: background, clutter, coherent RCS measurements, least-median squares, orthogonal-distance regression, outliers, radar cross section, RCS, RCS uncertainty

1. Introduction

By definition the radar cross section (RCS) of a target is the squared amplitude of the electric field scattered by a target located at infinity and illuminated by a plane wave. In practice, the reflected electric field is measured monostatically or bistatically at a large distance d between the target and the transmitting and receiving antennae, such that $kd >> 2\pi$, where k is the transmitted wavenumber. The *measured signal* S is the resultant of a theoretical electric field scattered by the target, fields that originate from the measurement environment, and distortions due to instrumentation nonlinearity, noise and rf interference. The measured complex electric field S can be written

$$S(r, \theta, b, \beta) = re^{i\theta} + be^{i\beta} + n + o, \qquad (1)$$

where r and θ are the amplitude and phase of the reflected electric field signal from the target (that could possibly include in-phase error signals), and b and β are the amplitude and phase of the electric field signal originating from the environment (mostly clutter, but may include calibration and instrumentation effects), n is noise and o represents the outliers, for example, rf interference that inevitably shows up in real data. The first term in eq (1a) describes a circle in the complex plane centered on the origin as θ varies from 0 to 2π, and the second term is a constant that moves the center of the theoretical circle into the complex plane. Figure 1 illustrates schematically the difference between *ideal* and *real* RCS measurements as modeled in eq (1). Note that if $b < 2r$ there are two values of the phase $\theta - \beta$ where the measured RCS yields the theoretical value, that is, $S = r$ at these points. These values can be determined from the ratio b/r.

The method of obtaining r and b that fits real data without special consideration of outliers has been studied and presented at AMTA 2002 [1]. We have shown that a number of techniques are available to complete such an analysis to produce the parameters with acceptable uncertainties. Various sources of er-

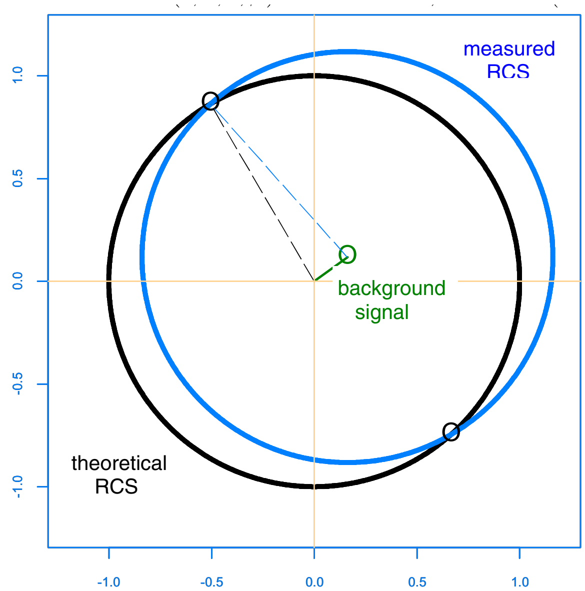

Figure 1. The normalized complex electric field r in *ideal* RCS measurements describes a circle centered on the origin, as the phase varies. The background signal $be^{i\beta}$ in *real* measurements S displaces the center of the ideal circle from the origin. By removing background signals from the measurements, RCS calibration accuracy is improved.

rors will affect the target and background signals differently. For example, we would expect drift to affect both the target and background similarly. The target signal, however, will also display variations due to illumination errors, since the actual nonplanar illumination varies with target position. Slight variations in target orientation due to motion should also contribute to the uncertainty in the target signal. In this study we use the orthogonal distance regression (ODR), developed at NIST, to obtain the parameters of the phase-dependent RCS data. We modify slightly eq (1) to obtain

$$(S_I - b_I)^2 + (S_Q - b_Q)^2 = r^2, \qquad (2a)$$

where $be^{i\beta} = (b_I, b_Q)$, and we follow the accepted usage in the RCS community) to denote the real and imaginary components of a complex quantity with the subscripts I and Q. The phase θ of r is given by

$$\tan\theta = \frac{S_Q - b_Q}{S_I - b_I}. \qquad (2b)$$

Equation (2a) does not depend explicitly on θ, which is unknown, and can be used to obtain r^2, the RCS of a calibration or an unknown target, in the presence of a complex signal b (due to the environment and the measurement system). Experimental realization of this approach is, however, challenging: we need to vary θ without significantly varying b and β. Variations in b and β, such that $\delta b << b$ and $\delta\beta << \delta\theta$, will be incorporated into the overall uncertainty of the measurement. In addition, our data must not be contaminated by a large number of outliers, or even a small number of extreme outliers. Least-squares analysis techniques lack robustness, in the sense that even a single outlier can significantly degrade the accuracy of the measurement parameters. Since RCS measurements are usually subject to a large number of outliers, this problem needs to be addressed explicitly.

2. LMS and ODR Analysis

In Figure 2, the real and imaginary components S_I and S_Q of the electric field data are constrained on a *data circle* of radius r, centered at (b_I, b_Q). The *measured* complex electric field S is expressed with respect to the origin, and the fractional measurement error is given by S/r. Obviously, if we can determine and remove the background signal b from the data, the measurement error can be significantly reduced.

Figures 2 and 3 show real data at 0.175 GHz that include a large number of outliers. The corresponding signal r and background levels b are also shown. In figure 2 the outliers were not eliminated from the data set before ODR was used to determine the parameters. In figure 3 we show the results when the outliers are eliminated using a least-median square analysis before ODR is applied. The difference is immediately apparent. The parameters and their uncertainties are summarized in Table 1.

Similarly, Figures 4 – 7 show real data at 2.8 GHz. In addition to outliers present in the real data, the large rf interference outliers seen in Figure 5 and 7 were introduced artificially to further test outlier dependence and detection. In figures 4 and 5 the outliers were not eliminated before ODR was used to determine the parameters; in figure 6 and 7 the combined LMS and ODR analysis produced nearly identical parameters to separate the background and target signals. The parameter and their uncertainties defining the circles in figures 4 – 7 are summerized in Table 1.

At 2.8 GHz approximately 1450 data points were taken as the Arrow III target moved a full 26 inches. Subsets of the data in 100 point sequences were isolated and analyzed using ODR with and without LMS prefiltering. In figure 8 we show the background and target parameters for these data sets. Several features are noteworthy: 1) the background level is higher than the target signal, 2) the background level shows only minor variations as the target is moving, 3) the target signal varies over a range of 1 dB, which is most likely due to variations in illuminations and target-mount interactions. Since the background level is relatively

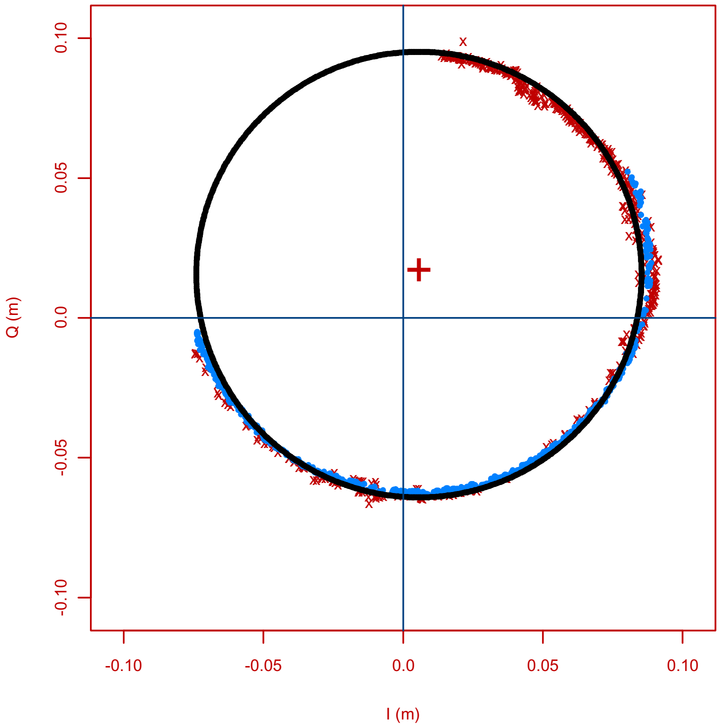

Figure 2. Phase-dependent RCS data at 0.175 GHz. The background (+) and the radius of the circle r were determined by ODR without outlier elimination. The RCS of the target is given by r^2. The crosses indicate the outliers, which however, were not eliminated from the data set.

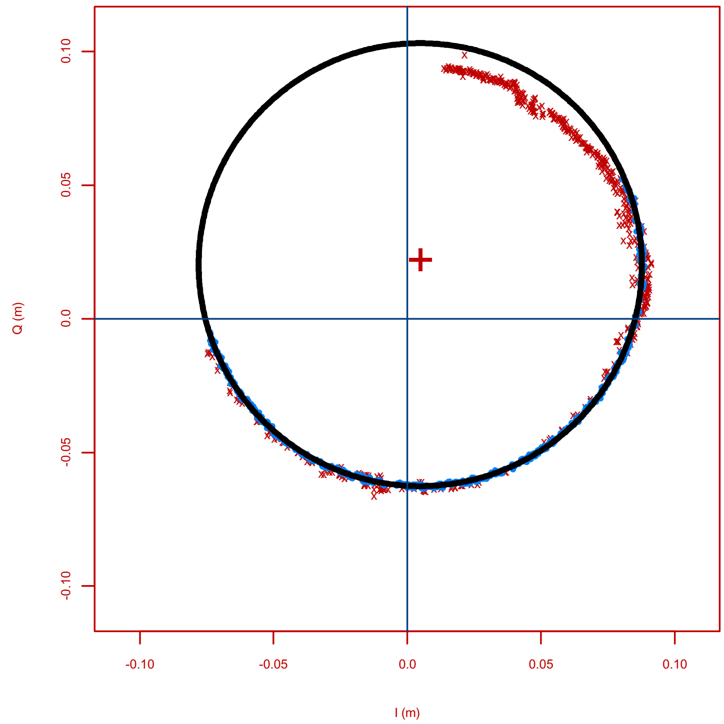

Figure 3. Phase-dependent RCS data at 0.175 GHz. The background (+) and the radius of the circle r were determined by removing the outliers using LMS and then applying ODR. The RCS of the target is given by r^2. The crosses indicate the outliers that were eliminated from the data set.

stable, we conclude that system drift is minimal. In addition, we note that the uncertainty bounds after LMS prefiltering that removes outliers decrease in all cases, as expected.

Finally, in figure 9 we show the full data set at 2.8 GHz using subsets of a short arc, long arc, long arc with rf interference, then 1/8, 1/4, 1/2 of the total number of points, finally, the full data set of 1450 points. After the outliers are eliminated, the long arc data sets show remarkable agreement. The uncertainty bounds of the parameters of the short arc data set are considerably greater that the other uncertainty bounds seen in figure 8. This is understandable since a short arc is underdetermined. Nevertheless, all parameters agree within 0.5 dB, showing that the data analysis technique presented here is robust and stable.

3. Conclusion

We have explored a phase-dependent RCS measurement and analysis technique that can reliably separate the background and target signals and outliers present in the data. The technique has been shown to be reliable even when the background level is above the target RCS. We demonstrated that elimination of outliers is an important first step in the analysis, since the inevitable presence of outliers significantly affects the results less robust least-squares techniques. The codes for this study were developed at NIST and are available to the RCS community on request.

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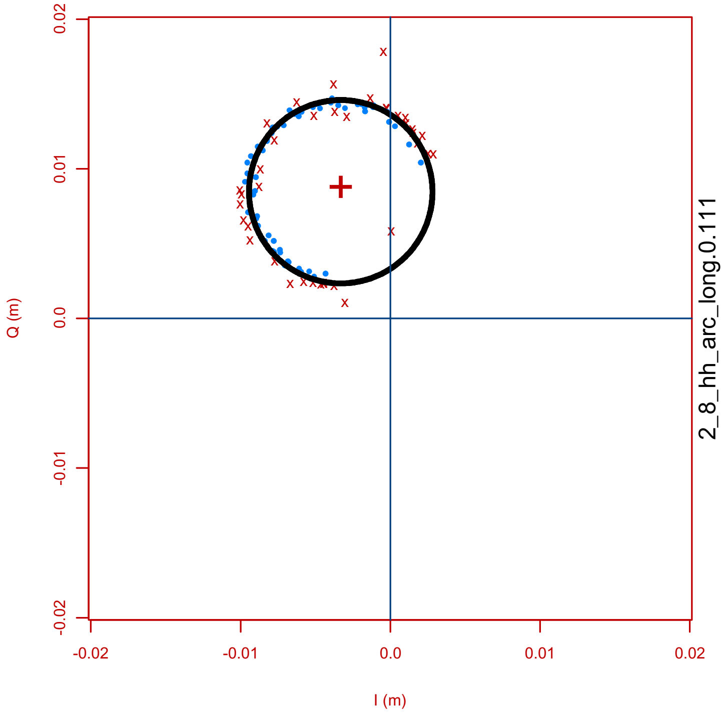


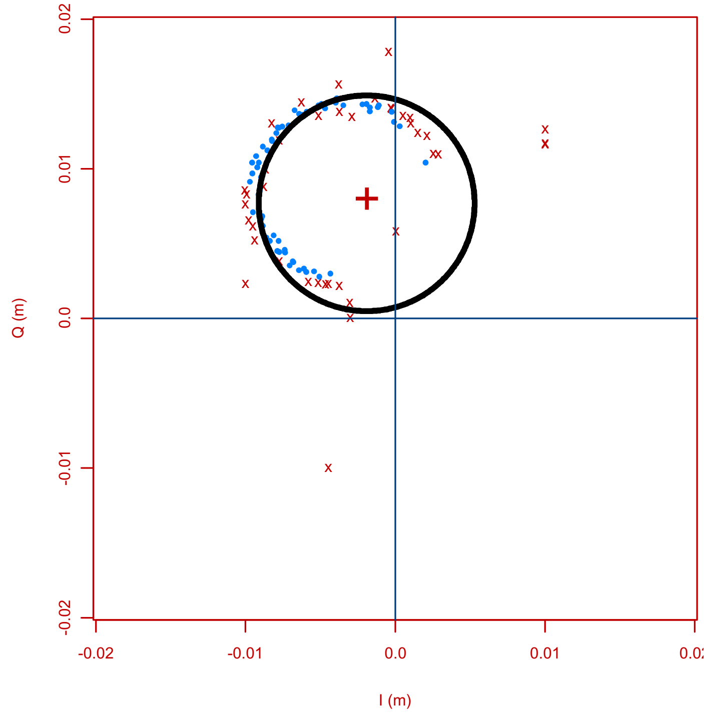
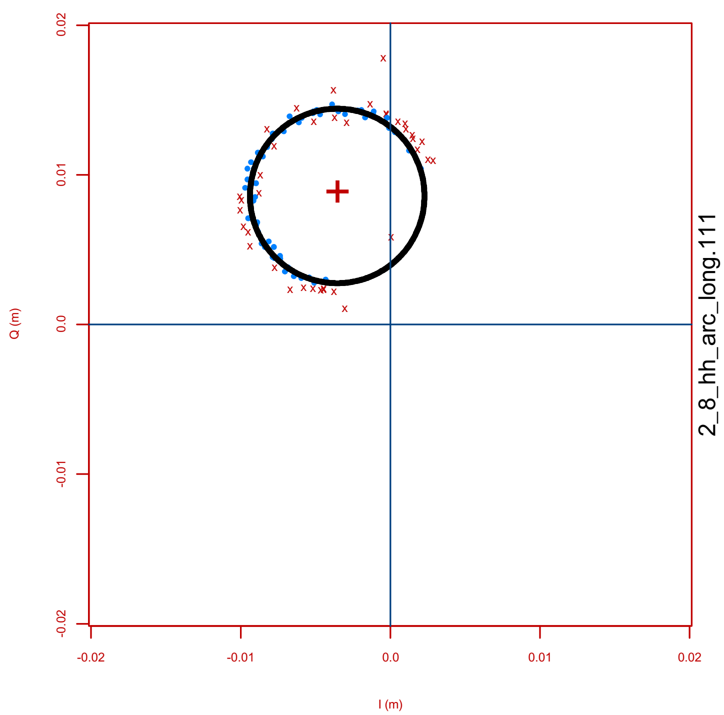


Figure 4. Phase-dependent RCS data at 2.8 GHz without rf interference. Outliers (x) were not eliminated before ODR was used to determine the background (+) and the radius of the circle.

Figure 5. Phase-dependent RCS data at 2.8 GHz with rf interference. Outliers (x) were not eliminated before ODR was used to determine the background (+) and the radius of the circle.



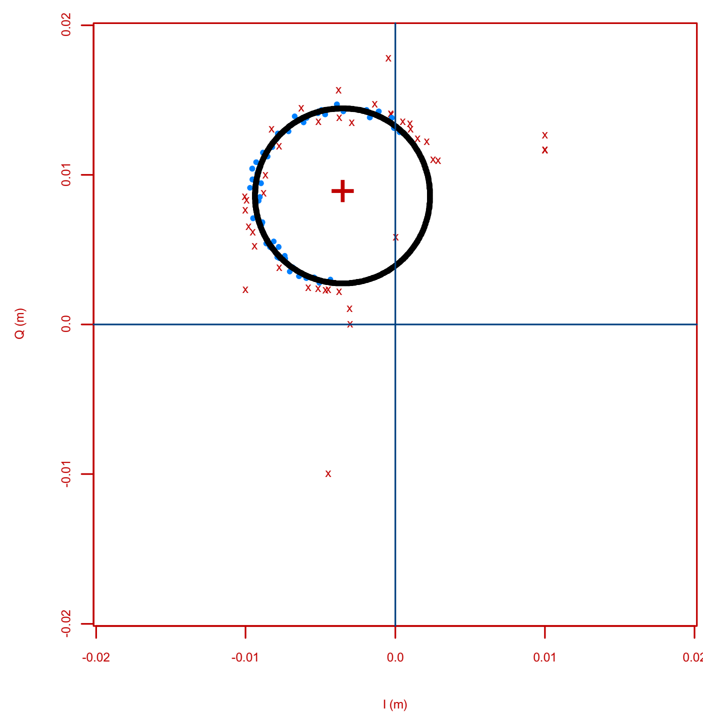


Figure 6. Phase-dependent RCS data at 2.8 GHz without rf interference. Outliers (x) were eliminated using LMS before ODR was used to determine the background (+) and the radius of the circle.

Figure 7. Phase-dependent RCS data at 2.8 GHz with rf interference. Outliers (x) were eliminated using LMS before ODR was used to determine the background (+) and the radius of the circle.

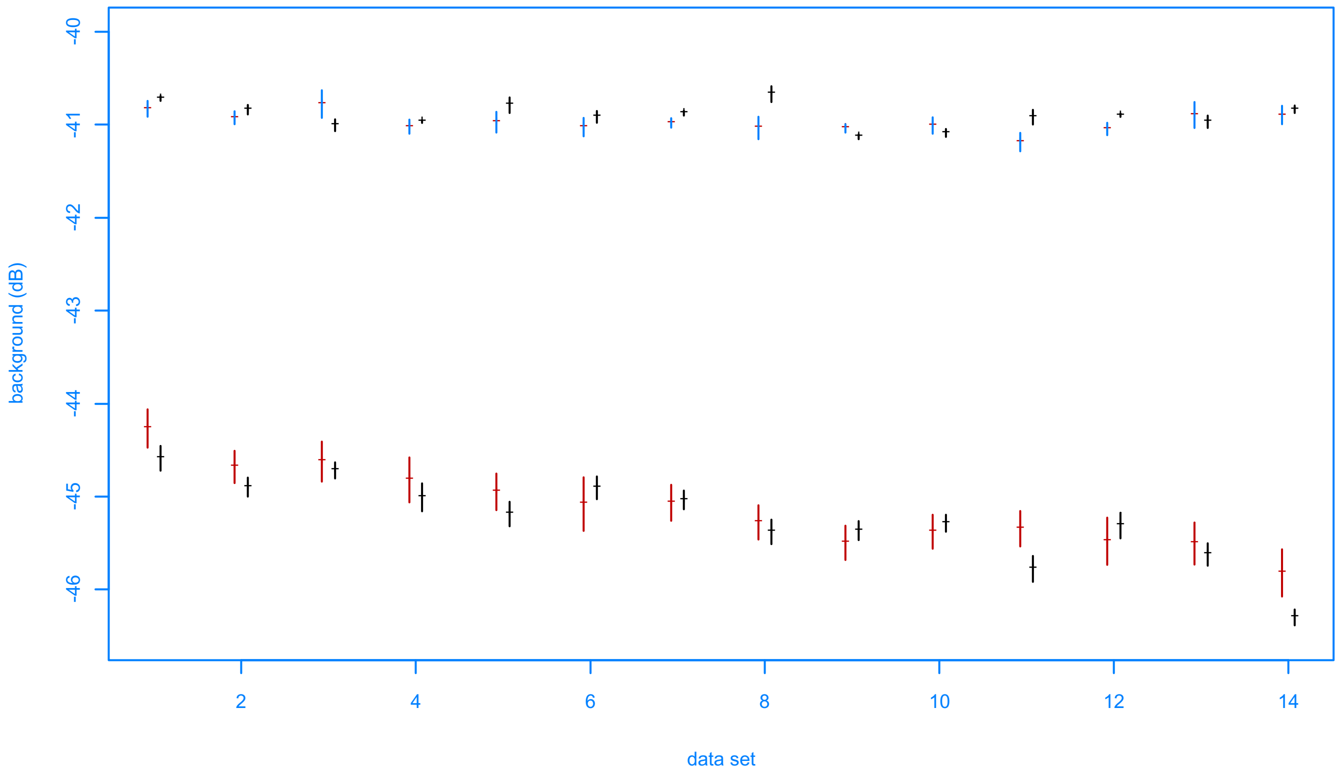


Figure 8. Phase-dependent RCS measurements at 2.8 GHz. Each of the background (top) and target (bottom) signals were obtained using 100 points in sequence. The total number of points in data set is 1450.

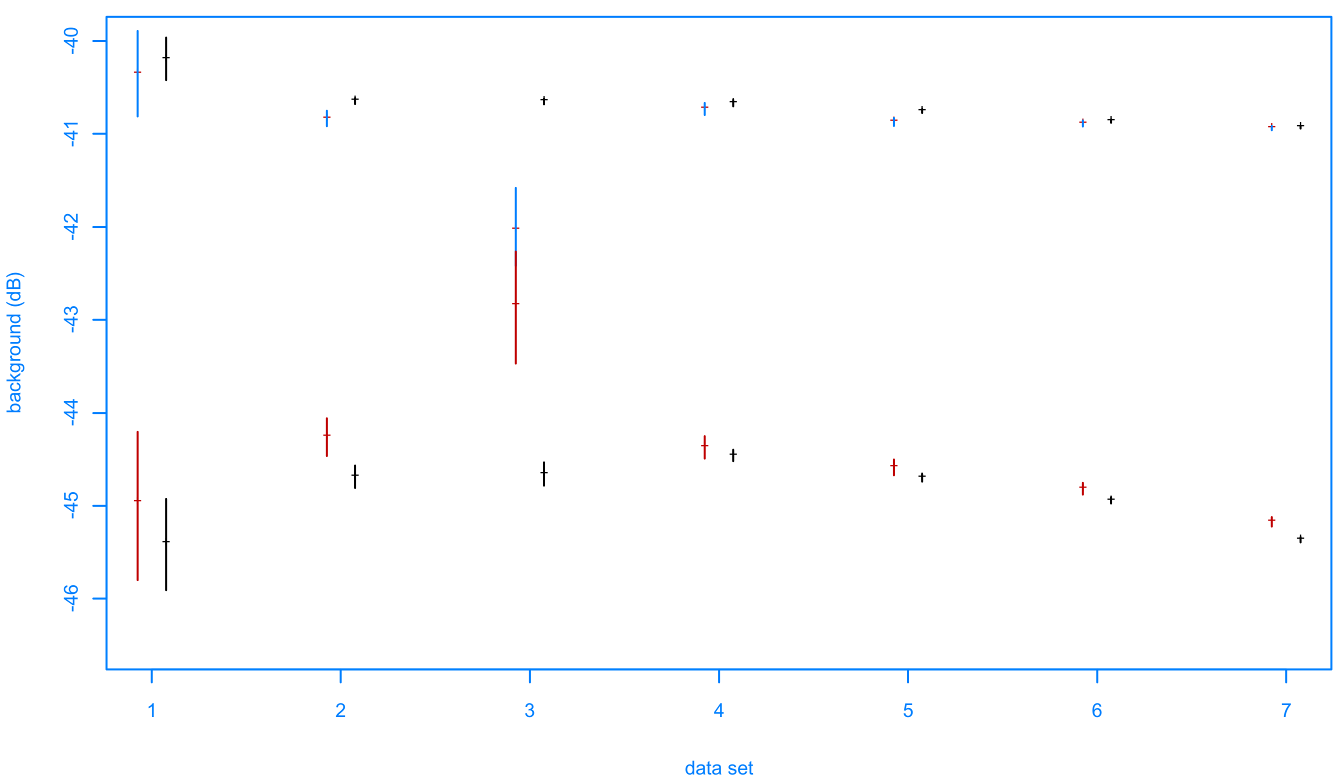


Figure 9. Phase-dependent RCS measurements at 2.8 GHz. The background (top) and target (bottom) signals were obtained using data points that comprise 1) a short arc, 2) a long arc, 3) a long arc with rf interference, 4) 1/8, 5) 1/4, 6) 1/2 of the full data set, and 7) the full data set. The total number of points in data set is 1450.

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